Space Track Launch System
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by
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1. Introduction

Perhaps the first use of a rotating tether to sling payloads into orbit was mentioned by Joseph A. Carroll (Carroll, J.A.,1986). Carroll suggested that a rotating sling on the surface of an airless body such as the moon might accelerate 10-20 kg payloads to orbital velocity. J. Puig-Suari, et el, (Puig-Suari, J., Longuski, J. M., & Tragesser, S. G., 1995) explored the technical feasibility of a tether sling for lunar and planetary missions. The requirements for the tapered tether and the energy demands for some representative tether sling facilities were evaluated. In a NASA publication, D.V. Smitherman, Jr. (Smitherman Jr., D. V., 2000) suggested using a spinning tether on top of a 50 km tall tower as a sling to launch cargo into space from earth. A major drawback to these concepts is the lateral force placed on the tower when the payload is released. The Space Track Launch System is similar to these concepts but with a few exceptions. Space Track uses a counterweight permanently attached to the end of the ribbon and rotational kinetic energy to launch the payload from the ribbon. In this manner, the launch load is transferred to the ribbon, keeping the dynamic load on the tower to a minimum. The ribbon can be inspected periodically and replaced if necessary.

The proposed Space Track Launch System is illustrated in Figure 1 below. Two tapered ribbons each of length l_r and cross sectional area A_x are attached to counter-weights (CW) with mass m_{cw}. A launch vehicle (LV) travels down the length of the ribbon and launches off the ribbon at the counterweight. The launch vehicle achieves a resultant velocity due to the gravitational, tangential, and centrifugal acceleration down the ribbon. The tower height is such that there is the smallest possible drag on the ribbon and tower.

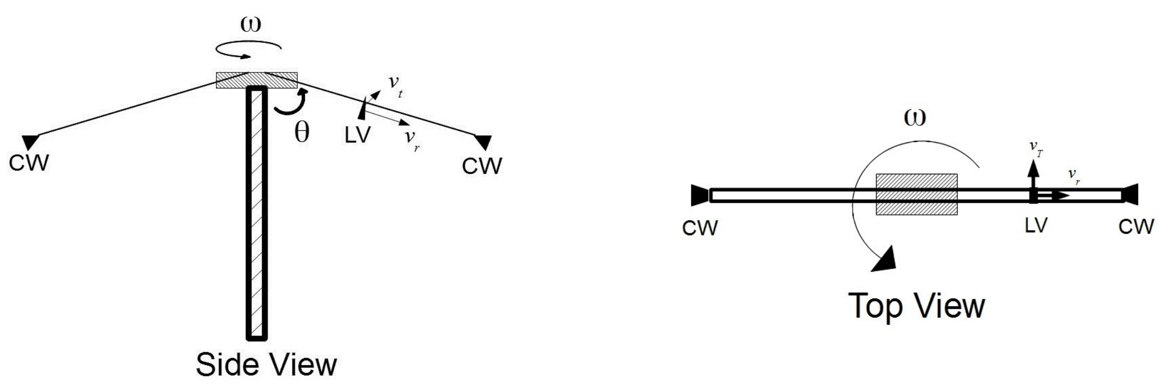

Figure 1. Space Track Launch System

In the following sections, the necessary equations are developed to determine the taper, area, and mass of the ribbon; the center of mass of the ribbon and counter-weight combination; the angular velocity; the resultant velocity of the launch vehicle at launch; the moment of inertia of the ribbon and counterweight; and the rotational kinetic energy for the Space Track Launch System. With these equations, a possible second generation launch system using a carbon nanotube (CNT) ribbon is evaluated. A first generation system using presently available materials is suggested.

2. Tapered Ribbon

The ribbon material, the tower height, and the altitude of the counterweight will determine the length of the ribbon and the angle it makes with the vertical. For an earth based launch system, atmospheric properties (mainly drag) force the tower height and counterweight altitude into the upper atmosphere. Although in some cases a uniform ribbon would work, a tapered ribbon is suggested because of the lengths involved.

The Space Track concept is similar to a conical pendulum. As shown in figure 2, the pendulum swings around an axis at an angle θ from the vertical. For a counterweight of mass m_{cw} revolving around an axis with radius r_{cw}, the tension in the ribbon at the counterweight is given by,

$$T_{CW}=\sqrt{(m_{CW}\,a_c)^2+(m_{CW}\,g)^2} \qquad (1)$$

where a_c is the centripetal acceleration equal to $\omega^2 l_r sin\theta$, ω is the angular velocity, l_r is the length of the ribbon, T is the tension in the ribbon, and g is the acceleration due to gravity.

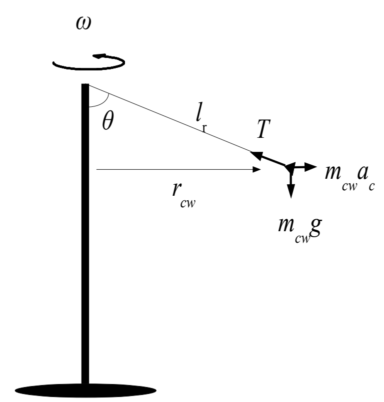

Figure 2. Conical Pendulum

For the Space Track Launch System, the width of the ribbon is kept constant to serve as a track for the launch vehicle. Therefore, the ribbon will taper in thickness from the rotating hub to the counterweight. The area of the ribbon at the counterweight is equal to the greatest allowable stress afforded by the ribbon. Therefore,

$$T_{CW}=\sigma\,A_{CW}=m_{CW}\sqrt{(\omega^2\,l_r\sin\theta)^2+g^2} \qquad (2)$$

where σ is the working tensile strength of the material.

For a ribbon of length l_r, the cross sectional area of the ribbon increases from the counterweight to the rotating hub. As such, the ribbon contributes a significant mass to the system. Therefore, to determine the angular velocity ω, the center of mass of the ribbon and counterweight combination must first be determined. The center of mass of a system of particles is given by,

$$l_{cm} = \frac{\sum_{i=1}^{N} m_i l_i}{\sum_{i=1}^{N} m_i} \qquad (3)$$

To find the center of mass of the ribbon, the ribbon is divided into 100 m segments. Beginning at the counterweight, the mass of each segment is determined by the density of the material times it's volume. The volume is given by multiplying the cross-sectional area by 100 m. The cross-sectional area at each segment is given by,

$$A_i = \frac{1}{\sigma} \sum_{i=0}^{N} m_i \sqrt{(\omega^2 l_{cm} \sin\theta)^2 + g^2} \qquad (4)$$

where A_i is the area at the mass segment (A_0 = area of the ribbon at the counterweight), m_i is the mass of the segment (m_0 is the mass of the counterweight), and l_{cm} is the distance from the top of the tower to each successive center of mass (at i = 0, l_{cm} = l_r). Using equation 3 above, the center of mass is recalculated after the mass of each segment is determined. It is the sum of the masses plus the mass of the counterweight and the distance to the center of mass from the top of the tower which determines the area for the next 100 m segment of ribbon. In this fashion, the center of mass of the ribbon and counterweight is determined. With the center of mass known, the angular velocity is given by,

$$\omega = \sqrt{\frac{g}{l_{cm} \cos\theta}} \qquad (5)$$

Unfortunately, the procedure requires the angular velocity to calculate the angular velocity. This requires an iteration process with an educated guess at the initial value.

For example, for a CNT ribbon 400 km in length at an angle of 78.5°, the center of mass could be half the length of the ribbon. This gives the initial angular velocity equal to 0.016 rad/sec. A spreadsheet was developed to handle the calculations and is included in the appendix. From the spreadsheet, for a CNT ribbon with ρ = 1300 kg/m^3 and σ = 25 GPa (1/6 of the ultimate tensile strength, 2003, Wei, C., Cho, K., & Srivastava, D.), the angular velocity is 0.0149 rad/sec, the center of mass is 223 km, and the area of the ribbon at the tower is 0.175 mm. With the angular velocity known, the velocity of the launch vehicle at launch v_L can now be determined.

4. Launch Velocity

It can be seen from Figure 1 that the velocity of the launch vehicle at launch is,

$$v_L = \sqrt{v_T^2 + v_R^2} \qquad (6)$$

where v_T is the tangential velocity and v_R is the radial velocity of the launch vehicle.

Figure 1 shows a launch vehicle (LV) accelerating down the ribbon. The tangential velocity of the launch vehicle at the launch point l_{pt} is,

$$v_T = \omega l_{pt} \sin\theta \qquad (7)$$

The work-energy relationship is used to determine the radial velocity v_R.

The radial force F_R on the launch vehicle at any point x along the ribbon is equal to the vector sum of the centrifugal force and the gravitational force at x. Therefore,

$$F_R = \sqrt{(m_{LV}\omega^2 x \sin\theta)^2 + (m_{LV} g)^2}$$

For long ribbons, the force due to gravity is small compared to the centrifugal force. Therefore, the radial force can be approximated by dropping the gravity term. This gives,

$$F_R \approx m_{LV}\omega^2 x \sin\theta$$

The work done on the launch vehicle from the rotating hub to the launch point is,

$$W = \int_0^{l_{pt}} F_R \, dx$$

Substituting F_R from above and integrating gives,

$$W = \left(\frac{1}{2}\right) m_{LV}\omega^2 l_{pt}^2 \sin\theta$$

The work is equal to the change in kinetic energy of the launch vehicle in the radial direction at launch. Therefore,

$$\left(\frac{1}{2}\right) m_{LV} v_R^2 = \left(\frac{1}{2}\right) m_{LV}\omega^2 l_r^2 \sin\theta$$

Solving for v_R gives,

$$v_R = \omega l_{pt} \sqrt{\sin\theta} \qquad (8)$$

Substituting v_T from equation 7 and v_R from equation 8 into equation 6 gives,

$$v_L = \omega l_{pt} \sqrt{\sin\theta(1+\sin\theta)}$$

where v_L is the velocity of the launch vehicle at launch.

To get the launch point l_{pt} and therefore, the launch velocity, the maximum peak acceleration must be considered. To get an approximate value for the launch point, it is necessary to ignore the effects of gravity and again make use of the work-energy relation. From equations 7 and 8 above the radial velocity, v_r, and the tangential velocity, v_t, are approximately equal. The acceleration due to gravity will be reintroduced to determine the total acceleration.

The centripetal acceleration is $\omega^2 r$ and the tangential acceleration is $2\omega v_r$. But since $v_r \approx v_t = \omega r$, the tangential acceleration is $2\omega^2 r$. Therefore, total acceleration is,

$$Ng = \sqrt{\left(\omega^2 r\right) + \left(2\omega^2 r\right) + g^2}$$

where N is the multiplier for the maximum acceleration (i.e., $N = 6$ for a 6*g* acceleration), g is acceleration due to gravity equal to 9.81 m/s^2, ω is the angular velocity, and r is the radial distance from the axis of rotation equal to $l_{pt} \sin\theta$. Solving the above equation for l_{pt} gives,

$$l_{pt} = \frac{g\sqrt{(N^2 - 1)}}{\sqrt{5}\omega^2 \sin\theta} \qquad (9)$$

For the CNT ribbon above, the launch point for a 6g acceleration is 119 km, the launch velocity is 2.5 km/s, the altitude of launch is approximately 130 km.

5. Kinetic Energy of Rotation

Kinetic energy is transferred to the launch vehicle and overcarriage from the Kinetic Energy of Rotation (*K.E.R.*) in the ribbon and counterweight. To determine the *K.E.R.*, the total moment of inertia of the ribbon and counterweight combination is calculated. The moment of inertia is given by,

$$I_{(r+cw)} = \sum_{i=0}^{N} \left(m_i r_i^2\right) \qquad (10)$$

The variables in equation 10 are already available from calculating the angular velocity. Therefore, by adding an additional column in the spreadsheet, the moment of inertia is readily available. The results are shown in the appendix. Since there are two ribbons and two counterweights, the total moment of inertia I_t of the system is $2I_{r+cw}$. Therefore, the $K.E.R.$ of the Space Track Launch System is,

$$K.E.R. = \frac{1}{2} I_t \omega^2$$

From the spreadsheet the $K.E.R.$ Is 1.3 x 10^{13} J and, for a launch mass of 100 ton (80 ton launch vehicle plus a 20 ton overcarriage), the kinetic energy of launch is 3.1 x 10^{11} J. To ensure dynamic stability of the tower and ribbon, it would be wise to launch another 100 ton mass 210 sec later. Therefore, the total energy of launch is 6.2 x 10^{11} J. With four 5 MW superconducting electric motors each having a mass of about 25 ton (2010, AMSC), it will take approximately 9 hours to restore the $K.E.R.$ to the tower. Note that the kinetic energy of launch is two orders of magnitude less than the $K.E.R.$ As such, the angle the ribbon makes with the tower changes slightly and the counterweight drops from approximately 70 km to 68 km.

6. Displacement of the Ribbon at Launch

Due to the Coriolis force on the launch vehicle and overcarriage and the fact that the ribbon is fixed at the tower and free at the counterweight, the ribbon will be displaced several thousand meters at the point of launch. This is very similar to the dynamic stability of the ribbon for the space elevator (2006, de Vries, J. page 44). From the Delft University of Technology final report, the displacement due to the Coriolis force is shown in figure 3 below and is given by,

$$\delta_{max} = l_{pt}\left(1 - \frac{l_{pt}}{l_r}\right)\left(\frac{2\omega v_R m_{LV}}{T}\right) \qquad (11)$$

where δ_{max} is the maximum displacement of the ribbon just prior to launch, l_{pt} is the distance from the tower truss at which launch occurs, l_r is the length of the ribbon, ω is the angular velocity, v_R is the radial velocity of the launch vehicle, m_{LV} is the mass of the launch vehicle and overcarriage, T is the tension in the ribbon, α is the displacement angle at the counterweight, and β is the displacement angle at the tower truss. Using the CNT ribbon parameters from above, the maximum displacement, δ_{max}, is approximately 10 km, the displacement angle at the tower, β, is about 2.1°, and the displacement angle at the counterweight, α, is approximately 4.8° for a launch velocity of 2.5 km/s (or a radial velocity of 1.8 km/s) and a launch mass of 100 ton.

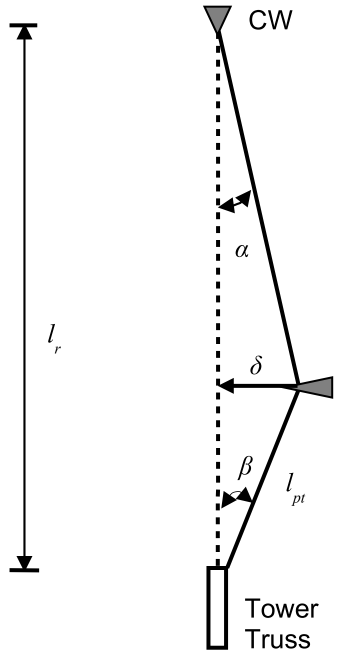

Figure 3. Displacement due to Coriolis Force

7.0 Payload to Low Earth Orbit

What is the maximum payload to low earth orbit? To simplify matters, chose a low earth orbit of 150 km, the same height as the tower. Therefore, there is no change in potential energy and since there is little drag at the launch altitude; the ideal rocket equation can be used directly with no modification. The ideal rocket equation is,

$$m_{pascent} = [\exp(\frac{\Delta v}{g I_{sp}}) - 1]\, m_{meco}$$

where $m_{pascent}$ is the mass of propellant required to achieve orbit, Δv is the change in velocity required for low earth orbit, g is the acceleration due to gravity, I_{sp} is the specific impulse, and m_{meco} is the mass of the launch vehicle at main engine cutoff.

The orbital velocity at 150 km is approximately 7,875 m/s. For a launch velocity of 2,899 m/s (which includes Earth's contribution), the Δv is 4,977 m/s. For a specific impulse of 330 sec (1992, Huzel, D.K. and Huang, D.H., page 380, Fig. 11-6) the mass of propellant required to achieve orbit is about 3.6 times the mass at main engine cutoff. For a gross liftoff mass of 80 ton, the mass at main engine cutoff is approximately 17.3 tons. This mass consist of the payload mass, inert mass of the launch vehicle which includes integral propellant tanks, and all remaining propellants necessary to achieve a stable orbit, for orbital maneuvering, and retro fire to return to Earth.

So, what is the mass of the launch vehicle at main engine cutoff in terms of the payload capability? This number is hard to quantify. An approximate value is determined by considering the only reusable launch vehicle, the space shuttle. From page 310 of the referenced text (2002, Jenkins, D. R.), select the last five OV-102 missions. Assume the weight of the orbiter on the launch pad is approximately equal to the weight of the orbiter at main engine cutoff. Divide this weight by the payload weight. This results in an average ratio of approximately 11. Therefore, the mass of the launch vehicle at main engine cutoff can be assumed to be approximately 11 times the payload mass. Using 17.3 tons as the launch vehicle mass at main engine cutoff gives a payload mass of approximately 1,500 kg. This is cargo to an orbiting space station or about 5 passengers to an orbiting space station.

8.0 First Generation System

The above analysis is based on the future development of the CNT ribbon. What is possible with present day materials? The Spectra® 2000 (2006, Spectra®) fiber will make an excellent first generation ribbon. It has a low density, 970 kg/m^3, a high strength, 3.5 GPa, bonds well with resins, and has the added advantage of being resistant to UV radiation, an attractive feature in the upper atmosphere. Therefore, the system consists of four Spectra® 2000 ribbons (two ribbons extending from opposite ends of the rotating truss). Each ribbon is 1 m wide. The ribbons serve as a track for the launch vehicle and overcarriage combination and are attached to a 20 ton counterweight.

The counterweight must remain above 70 km to avoid any air resistance. For a tower height of 150 km and a ribbon length of 160 km, this gives an angle of approximately 60° between the ribbon and the tower. Inserting an initial value for the angular velocity into the spreadsheet along with the ribbon parameters, gives an angular velocity of 0.0218 rad/sec, a moment of inertia of 2.84×10^{16} kg-m^2, a *K.E.R.* of 6.8×10^{12} J, and a thickness for the ribbon at the tower of 6 cm.

From equation 9, the launch point is 106 km for a 10g maximum acceleration giving a launch velocity of 3.4 km/s (which includes Earth's contribution). Using the same orbital altitude and engine parameters from above results in a 360 kg payload per launch vehicle to low earth orbit. Again, it would be wise to launch a second vehicle in approximately 144 seconds resulting in 720 kg to low earth orbit. From equation 9, the displacement of the ribbon at launch is approximately 310 m. With four 5 MW superconducting electric motors, it would take approximately 30 hours to restore *K.E.R.*

7. Summary

The Space Track Launch System is an all electric first stage launch system. A counterweight is permanently attached to the end of the ribbon and a liquid fueled fully reusable second stage launch vehicle is launched from the ribbon. The forces from the launch are several orders of magnitude less than the tension in the ribbon. As such, the stresses on the tower are greatly reduced. In the second generation system, a relatively small amount of the total kinetic energy of rotation is given up to the launch vehicle and can be restored in less than eight hours. A first generation system, built with present day materials, can launch two 20 ton launch vehicles 144 seconds apart and put approximately 720 kg of payload into a low earth orbit every thirty hours. Stronger materials under development today will lead to future generation systems with enhanced capabilities.

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Appendix

Mcw =	200000	
Omg =	0.0149	
Sig =	2.50E+10	
Rho =	1300	
Tht =	78.5	1.3700866667
Grv =	9.81	

Lx	MomInta	Mass Sum	MassDist	CM	AreaSeg	MassSeg	Omg/KER
4.00E+05	3.07E+16	2.00E+05	8.00E+10	4.00E+05	7.01E-04	9.11E+01	
4.00E+05	3.07E+16	2.00E+05	8.00E+10	4.00E+05	7.01E-04	9.11E+01	
4.00E+05	3.08E+16	2.00E+05	8.01E+10	4.00E+05	7.01E-04	9.12E+01	
4.00E+05	3.08E+16	2.00E+05	8.01E+10	4.00E+05	7.02E-04	9.12E+01	
4.00E+05	3.08E+16	2.00E+05	8.01E+10	4.00E+05	7.02E-04	9.12E+01	
5.00E+02	5.84E+16	8.95E+05	1.99E+11	2.23E+05	1.77E-03	2.30E+02	
4.00E+02	5.84E+16	8.95E+05	1.99E+11	2.23E+05	1.77E-03	2.30E+02	
3.00E+02	5.84E+16	8.95E+05	1.99E+11	2.23E+05	1.77E-03	2.30E+02	
2.00E+02	5.84E+16	8.95E+05	1.99E+11	2.22E+05	1.77E-03	2.30E+02	
1.00E+02	5.84E+16	8.96E+05	1.99E+11	2.22E+05	1.77E-03	2.30E+02	
0.00E+00	5.84E+16	8.96E+05	1.99E+11	2.22E+05	1.77E-03	2.30E+02	
							0.0148766016
							1.2966E+13

<u>Lx</u>
The distance from the rotating truss to the mass segment. The initial distance is lr. The second row is =A10-100, third row is =A11-100, etc.

<u>MomInta</u>
The moment of inertia. First row it is just the mass of the counterweight multiplied by its distance from the axis of rotation squared (=mcw *(A10*sin(tht))^2). Each successive row it is the mass of the segment multiplied by its distance from the axis of rotation squared (=B10 + G10*(A11*sin(tht))^2, =B11 + G11*(A12*sin(tht))^2, etc). The sum of the moment of inertia of each mass segment is added to that of the counterweight for a total moment of inertia.

<u>Mass Sum</u>
The running sum of the masses (=mcw, =C10+G10, =C11+G11, etc.)

<u>MassDist</u>
The mass of each segment times its distance for the rotating truss (=mcw*A10, =D10+G10*A11, D11+G11*A12, etc.)

<u>CM</u>
The center of mass of each successive mass segment (=D10/mcw, =D11/C11, =D12/C12, etc.)

AreaSeg
The area of the mass segment as given by equation 4 (=mcw/sig*SQRT((omg^2*E10*sin(tht))^2+grv^2, =C11/sig*SQRT((omg^2*E11*sin(tht))^2+grv^2, =C12/sig*SQRT((omg^2*E12*sin(tht))^2+grv^2, etc.).

MassSeg
The mass of the segment (=100*F10*rho, =100*F11*rho, =100*F12*rho, etc.).

Omg/KER
Recalculates the angular velocity once the center of mass is determined (=SQRT(grv/(E4010*cos(tht))) and calculates the kinetic energy of rotation (=B4010*omg^2) for both ribbons and counterweights.