Space Track Launch System
Second Stage Requirements

by
Jerry F. Fisher

Abstract

The Space Track Launch System consists of two stages. The first stage is a tall tower with rotating ribbons while the second stage is a liquid fueled cargo vehicle or passenger vehicle. This system is unique for several reasons. First, the first stage is all electric and can be used two or three times every day. The electric motors restore rotational kinetic energy to the ribbons in approximately six hours. Second, there are two sets of ribbons which allow one set to remain operational while the other is undergoing inspection and repair. Finally, for launch altitudes greater than 100 km, the second stage launch vehicle can take advantage of the gravity assist provided by the Earth resulting in significant propellant mass savings. This paper addresses the requirements for a second stage launch vehicle. A conceptual design is needed to help determine the load and tower base dimensions for the Space Track Launch System tower.

1. Introduction

The launch vehicle for the Space Track Launch System (STLS) makes up the second stage of a two stage system. The first stage is a tall tower with rotating ribbons (Fisher, J. F., 2007) and the second stage is a liquid fueled cargo vehicle or passenger vehicle. The tower (Fig. 1) will be 50-150 km in height. At the top of the tower, there is a rotating truss which supports four ribbons (two ribbons from each end of the truss) made of high strength fiber composites. The truss is powered by electric motors. Each ribbon is attached to a counterweight to provide stability and shift the center of mass further down the ribbon.

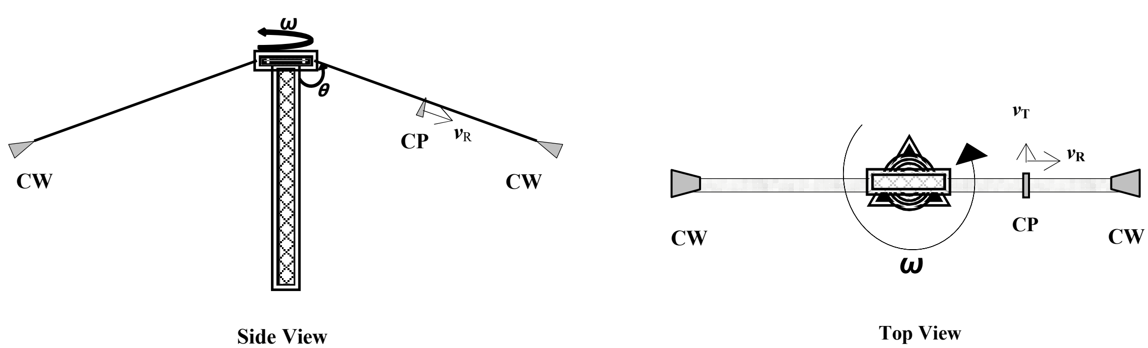

Figure 1. Space Track Launch System

The second stage in the launch system is a liquid fueled vehicle designed to launch from the rotating ribbons of the STLS. The launch vehicle attaches to an ejector which is attached to a carriage system. The carriage has several tapered rollers which rest on top of the ribbon. The carriage and launch vehicle travel down the ribbon and are accelerated by the centrifugal force resulting from the distance from the axis of rotation and by the contact force (Coriolis force) provided by the rotating ribbon. At a predetermined point along the ribbon, the ejector fires and the launch vehicle detaches from the carriage system and ribbon. Second stage operation begins when the liquid propellant rocket engines ignite.

The system is unique for several reasons. First, the first stage is completely electric and can be used two or three times every day. The electric motors restore rotational kinetic energy to the ribbons in approximately six hours. Second, there are two sets of ribbons which allow one set to remain operational while the other is undergoing inspection and repair. Finally, for launch altitudes greater than 100 km, the second stage launch vehicle can take advantage of the gravity assist provided by the Earth resulting in significant propellant mass savings.

The first generation tower will be approximately 50 - 100 km in height, use presently available composites for the tower and ribbons, and will launch vehicles massing 25 - 50 metric tons. The second generation tower will be 100 -150 km in height, use carbon nanotube ribbons, and launch vehicles massing less than 100 tons. If carbon nanotube ribbons are not available, presently available composite ribbons could be used. However, the system performance will suffer.

Since the tower will be the major cost driver, it is desirable to build only one tower for both first and second generation launch vehicles. The second generation system places the greatest burden on the tower in terms of load at the top of the tower, the dimensions of the tower base, and the dynamics of rotation. Therefore, a conceptual design of the second generation system will be done first, starting with the launch vehicle.

The launch vehicle is a reusable, liquid fueled second generation passenger or cargo vehicle designed for routine service to low earth orbit. Here, routine is defined as one launch every three days. Daily service to low earth orbit will require a small fleet of passenger or cargo vehicles. A conceptual design is needed to help determine the load and tower base dimensions for the STLS tower. The mass ratios of the Space Shuttle will be used to get an initial estimate for inert weight, landed weight, and gross liftoff weight for the second stage launch vehicle.

2. Space Shuttle Mass Ratios

The Space Shuttle is the only reusable orbiting vehicle currently in operation. The launch mass, the payload mass, and more importantly, the mass of the Space Shuttle at main engine cutoff (MECO) are well documented. Therefore, its mass ratios will be used to estimate the mass ratios for a future reusable orbiter. Using these mass ratios, conceptual designs for possible second generation orbital vehicles will be developed.

The structural ratio (R) for the Space Shuttle orbiter is about 2.8 (Henry, G. N., 2004). R is defined as the inert vehicle mass ($mass_{\text{inert}}$) divided by the payload mass ($mass_{\text{pyld}}$). As presently envisioned, the propellant tanks for the launch vehicle are internal to the structure. Therefore, to get a more accurate structural ratio, the external tank for the Space Shuttle has been added to its inert mass. This results in a structural ratio of 3.8. The Space Shuttle uses 1970s technology. Mass savings will result in new designs which use modern avionics, improved thermal protection systems, and composite structural components. However, for the purposes of an initial mass estimate for the conceptual designs, a structural ratio of 3.8 is adequate.

Another useful parameter is the propellant mass fraction (f_{prop}) which is defined as the propellant mass ($mass_{\text{prop}}$) divided by the sum of the propellant mass and the inert mass ($mass_{\text{prop}}$ + $mass_{\text{inert}}$). For the Space Shuttle orbiter, the propellant mass fraction is 0.16 (Henry, G. N., 2004). Including the external tank along with the inert mass gives a propellant mass fraction of 0.12. To summarize,

$$R = \frac{mass_{inert}}{mass_{pyld}} = 3.8 \qquad f_{prop} = \frac{mass_{prop}}{mass_{prop} + mass_{inert}} = 0.12$$

3. Launch Vehicle Mass Estimate

Second stage launch vehicle mass estimates are required to help determine an estimate for the tower load and dimensions. The mass estimates illustrate what could be possible with an electrically driven first stage launch system. For example, suppose the primary objective is to deliver six passengers to a low earth orbit space hotel and convention center. The payload, therefore, will consist of the crew, passengers, gear, and consumables.

The crew would consist of a pilot and a copilot. For safety reasons, each person would be required to wear a space suit during the flight to the space hotel. Newer concepts (Newman, D. J., Hoffman, J., Bethke, K., Carr, C., Jordan, N., Sim, L., Campos, N., Conlee, C., Smith, B., Wilcox, J. 2005) for spacesuits suggest that future spacesuits can be made much more comfortable and with a mass much less than the 90 kg Extravehicular Mobility Unit (EMU) used on the Space Shuttle. However, an EMU mass of 90 kg will be used for the purpose of this paper. At an average mass of 80 kg per person plus the 90 kg spacesuit, the total mass would be 1,360 kg for the six passengers and two crew members. Each person would require at least 105 kg of consumables which results in a mass of 840 kg. Adding the crew, passengers, gear, and consumables, the payload estimate is 2,200 kg. To include miscellaneous and personal items, the payload estimate is increased by one third. This results in a payload mass estimate of 3,000 kg.

With the payload determined, the inert mass for the launch vehicle can be derived using the structural mass ratio of 3.8 from above. This results in an inert mass of 11,400 kg. Using the propellant mass fraction of 0.12, the mass of OMS propellant required to establish orbit from MECO is about 1,555 kg. Therefore, the mass of the orbital vehicle at MECO is the inert mass plus the OMS propellant mass or 12,955 kg.

The second generation STLS is unique in that launch occurs at altitudes greater than 100 km and at velocities greater than 3.0 km/s. For example, if the tower height for a second generation system is 150 km and the carbon nanotube ribbon is 625 km long, then the velocity of the launch vehicle for a 6g launch from the ribbon is approximately 2,925 m/s at an altitude of 120 km. Due to the angle the ribbon makes with the tower, the launch velocity has a radial component of 486 m/s downward and a tangential component of 2,885 m/s. Due to the earth's rotation, a velocity of 408 m/s (latitude of 28.5^{o}) can be added to the tangential component. This makes the launch velocity from the ribbon 3,329 m/s. The orbital velocity at 120 km is 7,832 m/s. Therefore, the ΔV required to reach a 120 km circular orbit is about 4,503 m/s.

Using the ideal rocket equation, as shown below, the mass of propellant required and the gross liftoff mass (GLOM) can be estimated for several second stage systems. From the ideal rocket equation:

$$m_{main-prop} = \left[\exp\left(\frac{\Delta V}{gI_{sp}} \right) - 1 \right] m_f$$

where, $m_{main\text{-}prop}$ is equal to the mass of the main propellant required to reach 7,832 m/s, m_f is equal to 15,955 kg, the mass of the launch vehicle and payload at MECO, g is the acceleration due to gravity, and I_{sp} is the vacuum specific impulse for the liquid propellant rocket engines.

The GLOM for five different second stage systems is shown in table I below. These systems are representatives of high performance/very complex, medium performance/moderately complex, and low performance/low complex systems. The specific impulse was derived using NASA CEA program (Gordon, S. and McBride, B. J., 1994) optimizing the mixture ratio and assuming equilibrium conditions. Each system is pressure fed with a combustion chamber pressure of 500 psia, a nozzle expansion coefficient of 50:1, and a c* efficiency of 98%.

Table I. Initial Mass Estimates

System (Vac Isp)	LH/LF (461 s)	LH/LOX (443 s)	LOX/RP-1 (350 s)	99%H_2O_2/RP-1 (323 s)	90%H_2O_2/RP-1 (310 s)
Payload Mass	3,000	3,000	3,000	3,000	3,000
Inert Mass	11,400	11,400	11,400	11,400	11,400
OMS Prop Mass	1,555	1,555	1,555	1,555	1,555
Mass @ MECO	15,955 kg	15,955 kg	15,955 kg	15,955 kg	15,955 kg
Mass of Propellant	27,259	29,041	43,265	49,997	55,512
GLOM	43,214 kg	44,996 kg	59,220 kg	65,952 kg	71,467 kg

As shown in table I, even the low performance 90%H_2O_2/RP-1 system is under the 100 ton design limit for the STLS. In terms of mass, the STLS tower can be designed to accommodate a variety of second stage launch systems.

4. Gravity Assist

In the previous section the ideal rocket equation was used to estimate the mass of propellant required to reach a 120 km circular orbit at MECO. But, is this mass enough? To answer this question, the equations of motion and conservation of energy can be used to provide a first order approximation to the mass of propellant required.

Continuing with the example from above, the velocity of the second stage vehicle launched from the ribbon is approximately 3,329 m/s at an altitude of 120 km. The launch velocity has a radial component of 486 m/s downward and a tangential component of 3,293 m/s. At launch from the ribbon, the main rocket engines ignite to accelerate the vehicle into orbit.

The thrust (in Newtons) for each system is 1.5 times the GLOM. The mass flow rate is the thrust divided by the specific impulse. With the thrust and the mass flow rate, Newton's equations of motion can be used to give acceleration, velocity, and position as a function of time. At the end of the burn, the orbital energy is calculated using the known velocity and altitude. With little drag, conservation of orbital energy can be used to determine if there is enough velocity to achieve a stable 120 km circular orbit.

A FORTRAN program was written to handle the calculations and is included in the appendix. The results are shown in table II below.

Table II. Mass Estimates from Program SecondStage

System (Vac Isp)	LH/LF (461 s)	LH/LOX (443 s)	LOX/RP-1 (350 s)	99%H_2O_2/RP-1 (323 s)	90%H_2O_2/RP-1 (310 s)
Mass @ MECO (kg)	15,955	15,955	15,955	15,955	15,955
Mass of Prop (kg)	27,259	29,041	43,265	49,997	55,512
GLOM (kg)	43,214 kg	44,996 kg	59,220 kg	65,952 kg	71,467 kg
Thrust (N)	635,894	662,116	871,422	970,484	1,051,607
Flow Rate (kg/s)	94	102	169	204	231
Chg in Prop Mass	-12,331	-12,996	-21,070	-25,452	-29,217
New GLOM (kg)	30,883	32,000	38,150	40,500	42,250

As shown in table II, there is a significant discrepancy between the mass of the propellants required based on the ideal rocket equation and that required based on the equations of motion and conservation of energy. In fact, this illustrates one of the significant attributes of the STLS launch system. With a launch altitude of 120 km and a downward velocity of 486 m/s, the launch vehicle can take advantage of the gravity assist provided by the Earth. This gravity assist results in significant propellant mass savings. The greatest mass savings appears to be in the low performance/low risk second stage launch vehicle making the 90%H_2O_2/RP-1 launch vehicle an attractive second stage launch vehicle.

Reentry heating should not be a significant problem as long as the minimum altitude is kept above 100 km. Reentry of the Space Shuttle begins at approximately 120 km at a velocity of 7,800 m/s. The heat flux begins to increase significantly at an altitude of approximately 98 km (Ko, W. L., Gong, L., and Quinn, R. D., 2004). As long as the second stage launch vehicle stays above 100 km, it can take advantage of the gravity assist provided by the Earth. Therefore, the second stage vehicle has more than enough propellant to achieve a stable 120 km orbit.

With a little more burst from the main propulsion system, the second stage launch vehicle can perform a direct insertion burn and transfer directly to a 300 km orbit. Or, the OMS engine can fire to transfer the launch vehicle to a 300 km orbit to dock with the space hotel and convention center. The FORTRAN program included in the appendix can also be used to estimate the mass of propellant required for this maneuver. The preliminary dimensions for each system will be discussed in the next section.

5. Launch Vehicle Preliminary Dimensions

As currently envisioned, the second stage launch vehicle will attach to the carriage system at two load bearing points. The two propellant tanks will be designed as the load bearing structure for the launch vehicle. This results in a vehicle which is very similar to a catamaran sail boat in design. The preliminary design for an LOX/RP-1 second stage launch system is shown in figure 2 below. The launch vehicle has a central main cabin with two outrigger pontoons.

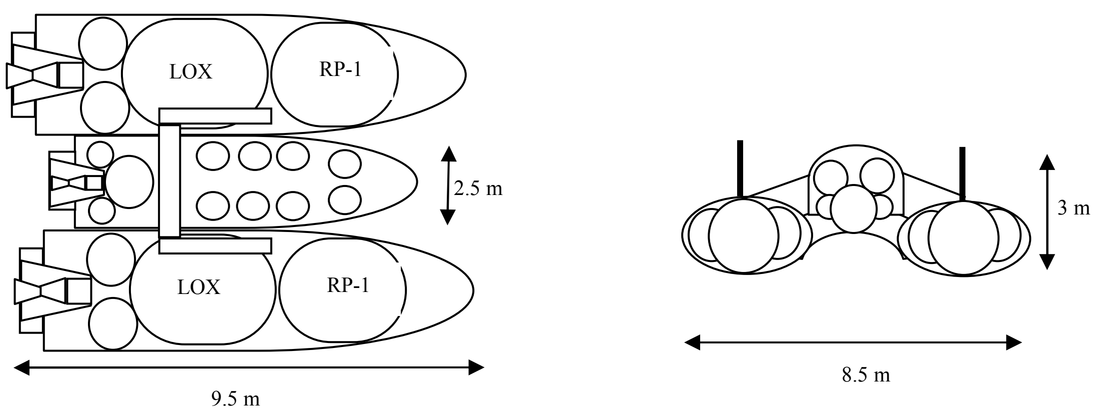

Figure 2. Second Stage Launch Vehicle

5.a Launch Vehicle Main Cabin

The central cabin is approximately 8.5 m long. This gives the two crew members and 6 passengers approximately 1 m^3 of volume each. Also, there is enough volume for the environmental control and life support system, storage containers and plumbing, the avionics, and consumables.

To the rear of the crew and passenger cabin is the housing for the parafoil recovery system, propellant for the orbital maneuvering system, thrust structure, gimbaling, and rocket engine. A parafoil recovery system was proposed for the X-38 crew recovery vehicle (NASA Facts, 2002). The X-38 was designed with a landing mass over 11,000 kg. Airborne Systems' (30K MegaFly, 2007) MegaFly ram air parafoil can deliver payloads up to 13,600 kg and has a glide ratio of 3.75:1. The ram air parafoil consist of five segments each weighing 110 kg. For ease of recover, the segments separate and can be repacked and ready for use again within eight hours. The entire package including parachute and containers weigh less than 900 kg. Therefore, the technology is presently available to support reentry vehicles comparable to the second stage launch vehicle of the STLS system.

The OMS propellant is 90%H_2O_2/RP-1 with an oxidizer to fuel ratio of 8:1. The preliminary dimensions for the orbital maneuvering system were derived using the procedures outlined in NASA SP-125 (Huzel, D. K. and Huang, D. H. 1967). SP-125 was published in 1967 at the height of the Apollo program. Almost all of the present knowledge pertaining to liquid fueled rocket engines was developed during the Apollo program. For the initial mass estimates, the H_2O_2 oxidizer tank is 1.3 m in diameter. The oxidizer tank has an internal positive expulsion diaphragm. With the diaphragm, the H_2O_2 monopropellant can be used for the reaction control system and to settle the propellants prior to ignition. The RP-1 tank and helium pressurant tank are both 0.8 m in diameter and are tucked in next to the oxidizer tank.

The thrust structure, gimbaling, and rocket engine are mounted behind the H_2O_2 oxidizer tank. The rocket engine provides a 65,900 N thrust. The combustion chamber is 70 cm long (including the catalyst screen) and 25 cm in diameter. The exhaust nozzle has an expansion ratio of 80:1 and is 1.6 meters in diameter at the exit plane. Together, the combustion chamber and exhaust nozzle are approximately 2.6 meters long. To protect the nozzle on reentry, the nozzle is retracted over the body flap. The central main cabin will be used when comparing all five different launch systems.

5.b Launch Vehicle Main Propulsion

For an initial preliminary dimensional analysis, a liquid oxygen and RP-1 pressure feed rocket engine will be used. This system represents the medium performance/moderate technology second stage launch system. The propellant mass from table I which results from the ideal rocket equation will be used. This gives a large design margin of over 21,000 kg as shown in table II. With this flexibility, either a smaller launch vehicle can be designed or a launch vehicle with a larger payload.

Each booster has a single pressure fed liquid oxygen and RP-1 rocket engine with a thrust of 436,000 N and an oxidizer to fuel ratio of 2.7:1. Using the mass estimate above for the propellant, the length of the booster is approximately 9.5 m, including the body flaps. The combustion chamber and exhaust nozzle are approximately 3.0 meters long. The exhaust nozzle has an expansion ratio of 50:1 and is 2.0 meters in diameter at the exit plane. Again, to protect the nozzles, they are retracted prior to reentry.

Oxidizer and fuel tank pressure is provided by two high pressure helium tanks. Each tank has a diameter of 1.3 meter and is connected to a pressure manifold at the base of the liquid oxygen tank. The helium passes through a heat exchanger connected to the exhaust nozzle. The heated helium then enters the oxidizer tank and fuel tank through a diffuser. The OMS engine is used to settle the propellants prior to ignition.

Similar procedures were used to estimate the dimensions of the remaining four types of launch systems. The results are shown in table III below.

Table III. Preliminary Dimensions for Second Stage Launch Vehicles

System	LH/LF	LH/LOX	LOX/RP-1	99%H_2O_2/RP-1	90%H_2O_2/RP-1
GLOM	43,214 kg	44,996 kg	59,220 kg	65,952 kg	71,467 kg
Length	13.5 m	15.8 m	9.5 m	8.5 m	8.5 m
Width	8.5 m	8.5 m	8.5 m	8.5 m	8.5 m
Height	3.0 m	3.0 m	3.0 m	3.0 m	3.0 m

The liquid hydrogen/ liquid fluorine (LH/LF) system is a higher performance system than the liquid hydrogen/liquid oxygen (LH/LOX) system resulting in the LH/LOX system being approximately 2.0 m longer but with similar GLOM. The lower performance H_2O_2/RP-1 systems are similar in size but with the 99% H_2O_2/RP-1 system having a lower GLOM. The initial dimensions for all of the second stage systems were based on each system having the same combustion chamber pressure, contraction ratio, and expansion ratio. Hence, each system can be further optimized by choosing the operating parameters a little more carefully. Further studies will address the optimization of future second generation launch systems.

6. Conclusion

This paper addressed the requirements for the second stage of a two stage system. A conceptual design of the launch vehicle is needed to help determine the load and tower base dimensions for the Space Track Launch System tower. The GLOM for the second stage launch vehicle ranges from 43 to 72 metric tons. The fuel requirements were based on the ideal rocket equation. By using the gravity assist provided by the Earth, the fuel requirements can be significantly reduced resulting in either a smaller second stage vehicle or a larger payload. The STLS tower will be designed to accommodate a second stage launch vehicle with the physical dimensions of 16.0 m long by 8.5 m wide by 3.0 m high. The ribbon, carriage, and counterweight will be designed to handle a maximum mass of 72 tons.

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APPENDIX

```
      PROGRAM SecondStage
! This program determines the orbital parameters of the second stage at MECO.
! The second stage is powered by 2 LOX/RP-1 rocket engines.
! The thrust and mass flow rate inputs
! are variable to help determine the optimum configuration. The program starts out by
! inputing the altitude, the radial velocity, and the tangential velocity derived from
! a companion program for 1st stage operation. It then goes through an iteration step
! to determine altitude, velocity, and range as a function of time.
!
      IMPLICIT NONE
      REAL ZR,THV, R
      REAL VT, VR, VEL, MI, GLOM, MDOT, FT
      REAL GM, AR, AT, DELT
      REAL TIME, OE
      REAL, PARAMETER:: PI=3.1715927, DTR=0.0174532, MU=3.986E14
      REAL, PARAMETER:: RE=6.378E6
      INTEGER STEP
!
! ZR is the radial altitude in meters.
! THV is the thrust angle between the radial and the tangential directions in degrees.
! R is the radial distance from the center of the Earth in meters.
! VT is the tangential velocity in m/s.
! VR is the radial velocity in m/s.
! VEL is the resultant velocity in m/s.
! MI is the insertion mass for the second stage vehicle in kg.
! GLOM is the gross liftoff mass in kg.
! MDOT is the mass flow rate in kg/s.
! FT is the thrust in Newton.
! GM is the acceleration of gravity in m/s^2.
! AR is the radial acceleration in m/s^2.
! AT is the tangential acceleration in m/s^2.
! DELT is the incremental time in sec.
```



```
! TIME is the total elapsed time in sec.
! OE is the specific orbital energy in J/kg.
! PI is pi.
! DTR is the conversion constant from degree to radian.
! MU is Earth's gravitational constant equal to 3.986x10^14 m^3/s^2.
! RE is the radius of Earth equal to 6.378x10^6 m.
! STEP is a counter to print results every 10 steps.
!
      PRINT*, 'Tangential and radial velocities (m/s).'
      VT=3701; VR=-486.; PRINT*,VT,VR
      PRINT*,'GLOM(kg) of the 2nd stage.'
      GLOM=42250.;PRINT*,GLOM
      PRINT*,'The insertion mass of the 2nd stage.'
      MI=15955;PRINT*,MI
      PRINT*,'The mass flow rate(kg/s).'
      MDOT=231.;PRINT*,MDOT
      PRINT*,'The thrust(N).'
      FT=1051607.;PRINT*,FT
      PRINT*, 'The thrust angle (deg).'
      PRINT*, 'An angle of 0 means that all thrust is in the tangential direction.'
      PRINT*, 'An angle of 90 means that all thrust is in the radial direction.'
      THV=33.; THV=THV*DTR; PRINT*, THV/DTR, THV
      PRINT*,'The altitude(m) at which second stage operation begins.'
      ZR=120000.;PRINT*,ZR
      PRINT*, 'The time increment (DELT in seconds)?'
      DELT=1.;PRINT*,DELT
!
```

The while loop will continue until the gross liftoff mass is equal to the insertion mass, i.e. until all of the propellant is spent. In the first IF statement, the thrust angle is reduce by 5.5 degree each loop after the radial velocity becomes greater than zero. Recall, the radial velocity has an initial negative value when launched from the ribbon. The next IF statement prevents the thrust angle from going below .1 radian or 5.7 degree. This parameter can be changed if the user wants to experiment with different thrust angles.

```
!
      R=ZR+RE
      TIME=0
      STEP=10
      DO WHILE (GLOM .GT. MI)
       IF (VR .GT. 0) THV=THV-5.5*DTR
       IF (THV .LE. .1) THV=.1
!
```

The acceleration due to gravity GM is dependent on the radius R from the center of the Earth. The launch vehicle is launched at an altitude of 120 km. Therefore, there is little drag on the vehicle. The radial acceleration AR is equal to the thrust FT in the radial direction divided by the vehicle mass at that moment in time minus the difference between the acceleration due to gravity GM and the centripetal acceleration VT^2/R. The tangential acceleration is just the thrust FT in the tangential direction divided by the vehicle mass at that moment in time. The imbedded while loop keeps the acceleration below 3g.

```
!
      GM=MU/R**2
      AR=FT*SIN(THV)/GLOM-(GM-VT**2/R)
      AT=FT*COS(THV)/GLOM
      PRINT*, AR, AT, SQRT(AR**2+AT**2), GM, GM-VT**2/R
      DO WHILE (SQRT(AR**2+AT**2) .GT. 30.)
        FT=FT-.01*FT
        MDOT=MDOT-.01*MDOT
        AR=FT*SIN(THV)/GLOM-(GM-VT**2/R)
        AT=FT*COS(THV)/GLOM
      END DO
!
```

A new radial distance and radial velocity is calculated based on the previous radial velocity and the new acceleration. Also, a new tangential velocity is calculated based on the previous tangential velocity and new tangential acceleration. The resultant velocity is determined and the vehicle mass is reduce due to the loss of propellant mass over the one second time interval.

```
!
      ZR=ZR+VR*DELT+.5*AR*DELT**2
      VR=VR+AR*DELT
      R=ZR+RE
      VT=VT+AT*DELT
      VEL=SQRT(VR**2+VT**2)
      GLOM=GLOM-MDOT*DELT
      IF (STEP .EQ. 10) THEN
        PRINT*, TIME, ZR, R
        PRINT*, TIME, VR, VT, VEL
        PRINT*, TIME, FT, THV/DTR, GLOM, MDOT
        STEP=0
        PAUSE
      ENDIF
      IF (ZR .LT. 30000.) THEN
        PRINT*, 'ZR is less than 30 km.'
        PAUSE
      ENDIF
      STEP=STEP+1
      TIME=TIME+DELT
    END DO
!
```

After all of the propellant is spent, the specific orbital energy is calculated using the kinetic and potential energy of the vehicle. Due to conservation of energy, the specific orbital energy is constant unless more thrust is applied. The loop will continue as long as the altitude ZR is greater than 40 km.


```
!
        STEP=10
        OE=.5*VEL**2-MU*(1/R-1/RE)
        DO WHILE (ZR .GT. 40000.)
          IF (STEP .EQ. 10) THEN
            PRINT*, TIME, ZR, R
            PRINT*, TIME, VR, VT, VEL
            PRINT*, TIME, OE
            STEP=0
            PAUSE
          ENDIF
!
```

The only acceleration on the vehicle is that produced by gravity and the centripetal acceleration which are both radial. These accelerations must balance if the vehicle is to stay in orbit. A stable orbit is hardly ever circular. However, it is possible to get close to a circular orbit. Eventually, the orbit will decay due to drag. But, for the purpose of this paper, the acceleration due to drag is ignored.

```
!
         AR=VT**2/R-GM
         ZR=ZR+VR*DELT+.5*AR*DELT**2
         VR=VR+AR*DELT
          R=ZR+RE
          VEL=SQRT(2*(OE+MU*(1/R-1/RE)))
          VT=SQRT(VEL**2-VR**2)
          GM=MU/R**2
          STEP=STEP+1
          TIME=TIME+1
        END DO
        END PROGRAM
```

