Fisher Space Systems, LLC

Space Track Launch System
Proof of Concept

Addendum B: Guy Wires for Tall Towers

by
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B1.0 Introduction

This addendum discusses the possibility of using guy wires to help stabilize the proof of concept tower for the Space Track Launch System (Fisher, J.F., 2012). It will be shown that Kevlar and Spectra® guy wires of constant cross sectional area are not practical for tall towers exceeding 20-30 km. With presently available materials, the only way to reach the 25 km height requirement for the proof of concept tower is to taper the cable.

Section B2.0 of the addendum derives the equation for a hanging cable with constant diameter and a constant horizontal component of the tension. Section B3.0 of the addendum uses the equation to reveal that a constant diameter cable is impractical for even the 25 km proof of concept tower. In section B4.0 of the addendum, a relatively simple spreadsheet is used to demonstrate that a tapered cable using presently available materials can theoretically meet the 25 km height requirement for the proof of concept system and possibly, even taller towers in the 50-150 km range. A conclusion is presented in section B5.0 of the addendum.

B2.0 The Catenary

The following solution for the hanging cable is from The Differential Equations Problem Solver, Vol 1 (REA, 1978). The symmetry of the problem is such that the horizontal component (H) of the tension (T) is assumed to be constant while the vertical component (V) varies with the weight of the cable. By choosing a point (P) on the cable (figure B1), the static dynamics of the cable is defined and the height of the cable as a function of the distances down range is determined by integrating the variables.

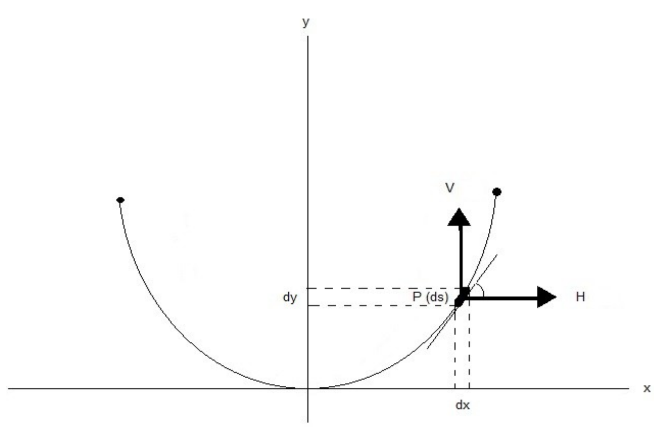

Figure B1. The Hanging Cable

With reference to figure B1 above,

$$\tan\theta = \frac{V}{H} = \frac{dy}{dx}$$

where θ is the angle of the tangent with the horizontal at point P, V is the vertical component of the tension (T), H is the horizontal component, ds is an element of the cable, and dx and dy are the change in x and the change in y respectively.

If w is the weight per unit length of the cable (N/m) then w ds is the weight of element ds. Therefore, the change in the vertical component of the tension is,

$$dV = w\,ds = \frac{w\,dx}{\cos\theta}$$

The vertical component of the tension (V) is equal to H tan θ, therefore,

$$d\left(H\tan\theta\right) = w\frac{dx}{\cos\theta}$$

The horizontal component (H) of the tension is assumed constant and therefore, it can be brought outside of the derivative. This gives,

$$H\frac{d}{dx}\tan\theta = \frac{w}{\cos\theta}$$

Taking the derivative of the tangent and using the chain rule gives,

$$H\,sec^2\theta\,\frac{d\theta}{dx} = \frac{w}{\cos\theta}$$

Simplifying by substituting $1/\cos^2\theta$ for $\sec^2\theta$ and then multiplying through by $\cos^2\theta$ gives,

$$\frac{H}{\cos^2\theta}\frac{d\theta}{dx} = \frac{w}{\cos\theta}$$

$$H\frac{d\theta}{dx} = w\cos\theta$$

Separating variables and integrating gives,

$$\frac{d\theta}{\cos\theta} = \frac{w}{H}dx$$

$$\int_0^{\theta} sec\,\theta\; d\,\theta = \frac{w}{H}\int_0^{x} dx$$

$$\ln|sec\,\theta + \tan\theta| = \frac{wx}{H}$$

$$sec\,\theta + \tan\theta = e^{\frac{wx}{H}}$$

The objective at this point is to get the left hand side of the equation in terms of only tan θ while keeping the right hand side of the equation as a function of x. The change in y divided by the change in x (dy/dx) is equal to tan θ and thus, integrating dy/dx will give y as a function of x. Be forewarned. What follows is an extensive use of trigonometric identities.

The half angle formula for the tangent is given below.

$$sec\,\theta + \tan\theta = \tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$$

From the half angle formula, additional trigonometric identities, and the fact that cot θ is equal to 1/tan θ , it follows that,

$$\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right) = e^{\frac{wx}{H}} \quad \text{and} \quad \cot\left(\frac{\theta}{2}+\frac{\pi}{4}\right) = e^{\frac{-wx}{H}}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\,\tan\beta} \quad \text{and} \quad \cot(\alpha+\beta) = \frac{1-\tan\alpha\,\tan\beta}{\tan\alpha + \tan\beta}$$

Substituting θ/2 for α and π/4 for β above, finding the common denominator, multiplying, and canceling everything out gives,

$$\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right) - \cot\left(\frac{\theta}{2}+\frac{\pi}{4}\right) = \frac{4\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

Using yet another trigonometric identity and replacing t with θ/2 gives,

$$\tan 2\text{t} = \frac{2\tan t}{1-\tan^2 t}$$

$$\frac{4\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = 2\tan\theta$$

and thus,

$$\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)-\cot\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=2\tan\theta$$

$$e^{\frac{wx}{H}}-e^{\frac{-wx}{H}}=2\tan\theta$$

$$\tan\theta=\frac{1}{2}\left(e^{\frac{wx}{H}}-e^{\frac{-wx}{H}}\right)=\sinh\left(\frac{wx}{H}\right)$$

But tan θ is also equal to dy/dx. As a result,

$$\frac{dy}{dx}=\sinh\left(\frac{wx}{H}\right)$$

Integrating the hyperbolic sine gives,

$$y=\frac{H}{w}\cosh\left(\frac{wx}{H}\right)+C$$

Using y = 0 at x = 0 gives the constant of integration as,

$$C=-\frac{H}{w}$$

Substituting the constant into the equation gives the final height y of the cable as a function of down range distance x as,

$$y=\frac{H}{w}\left(\cosh\left(\frac{wx}{H}\right)-1\right) \quad \text{(B1)}$$

where H is the horizontal component of the tension, w is the linear weight (N/m), and x is the downrange distances from the anchor. Both H and w are assumed to be constant.

B3.0 Guy Wires for the Proof of Concept Tower

For the 25 km proof of concept tower, a maximum horizontal load of about 10^7 N occurs at the top of the tower due to the torque produced when starting or stopping the rotation and recharging after the launch. There is also a lateral force of about 10^6 N when the counterweight swings south. From equation B1, using 10^7 N as the horizontal load, H, and setting the linear weight, w, at about 200 N/m (Kevlar 49, ρ = 1440 kg/m^3, σ_w = 1.2 GPa (SF = 3), radius of cable = 7 cm), and advancing the downrange distance, x, in 1 km increments produces the graph shown in Figure B2 below.

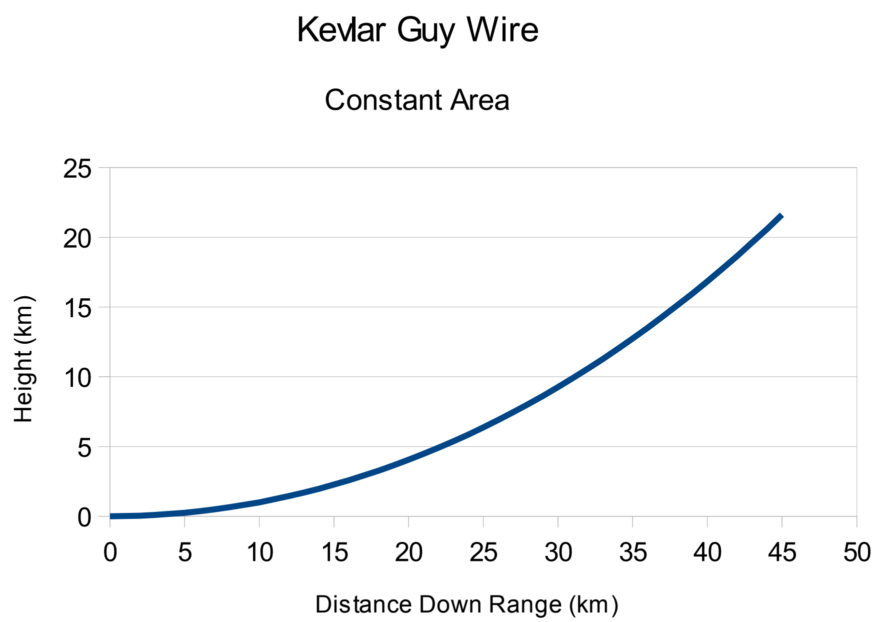

Figure B2. Kevlar Cable

As designed, the maximum height at which the Kevlar cable can reach before it exceeds the stress limit is about 22 km. As a quick check, the characteristic length of a vertically hanging Kevlar cable is about 255 km. With a diameter of 14 cm and a maximum load of 1.0 x 10^7 N, it would take about 70 km of cable to equal a load of 10^7 N leaving 39 km (SF=3) at the top to support the load. Therefore, a height of 22 km is not unreasonable for a constant area cable.

However, with a diameter of 14 cm, the cable already weights over 1000 metric tons and is over 50 km long. Obviously, manufacturing, transporting, and erecting such a cable would be impossible. But, what about using a Spectra® guy wire?

From equation B1 above, with a horizontal load, H, of 10^7 N and a linear weight, w, of 101 N/m (Spectra® 2000, ρ = 970 kg/m^3, σ_w = 1.2 GPa, area = 1.07 x 10^{-2} m^2, and a radius of 5.8 cm), the down range distance for a Spectra® cable is about 70 km (figure B3).

As designed, the Spectra® cable can reach 28 km in height before reaching its stress limit. As a quick check, the characteristic length for a vertically hanging Spectra® 2000 cable is about 342 km. At a radius of 5.8 cm, it would take about 81 km of cable to equal a vertical load of 8.1 x 10^6 N. This leaves about 33 km (SF=3) to support the load. Therefore, a height of 28 km is not unreasonable.

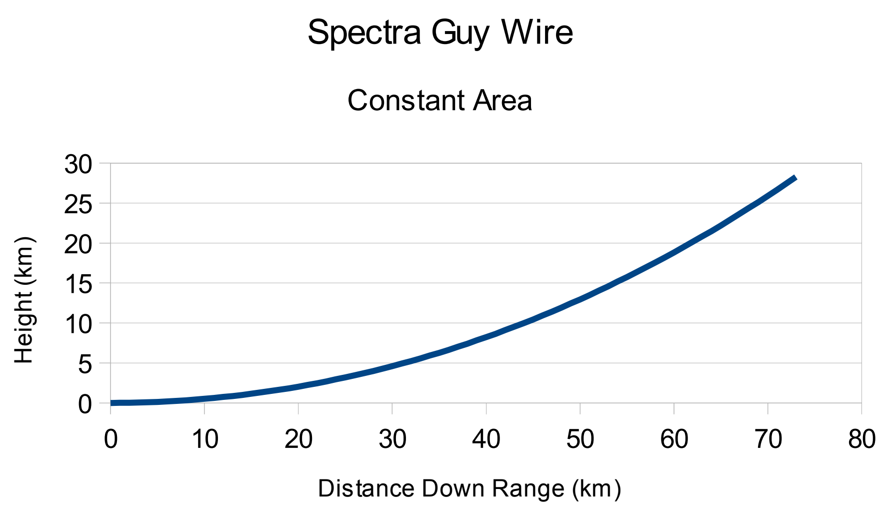

Figure B3. Spectra® Cable

For the proof of concept system, the cable would be about 74 km long and, with a diameter of 11.6 cm, has a mass of about 765 metric tons. Once again, manufacturing, transporting, and erecting such a cable would be extremely difficult. The solution is to taper the cable and build the cable in layers from the top down.

B4.0 Tapered Guy Wire

For a tapered guy wire, the wire is divided into 100 meter long by 50 cm wide sections. The area of each section is determined by the tension in the cable at that point. The mass of the cable can be determined and thus its contribution to the load. Summing the load for each section gives the total vertical component of the tension. The process continues until reaching the desired height.

For example, at y = 0 and θ = 0, the tension is equal to the horizontal load and peaks about 10^7 N. With a safety factor of 3, the area for the Kevlar cable would be about 10^{-2} m^2 and the weight for this section of cable would be about 14 kN. The angle the end of the cable makes with the horizontal is approximately 0.1° and its thickness is about 2 cm. The new vertical component of the load and the constant horizontal component of the load are used to find a new tension. That tension is used to find a new cross sectional area for the next 100 m section of cable and the process is repeated until reaching the desired height. A spreadsheet is used to add up all of the 100 m sections and plot the results. The results are shown in figure B4 below.

For the proof of concept tower, the desired height is 25 km. At this height, the tension in the cable is about 1.2 x 10^7 N. It is 2.3 cm thick, has an area of 1.2 x 10^{-2} m^2, and a mass of 676 metric tons. The cable makes a 31° angle with the top of the tower and has a down range distance of 40 km.

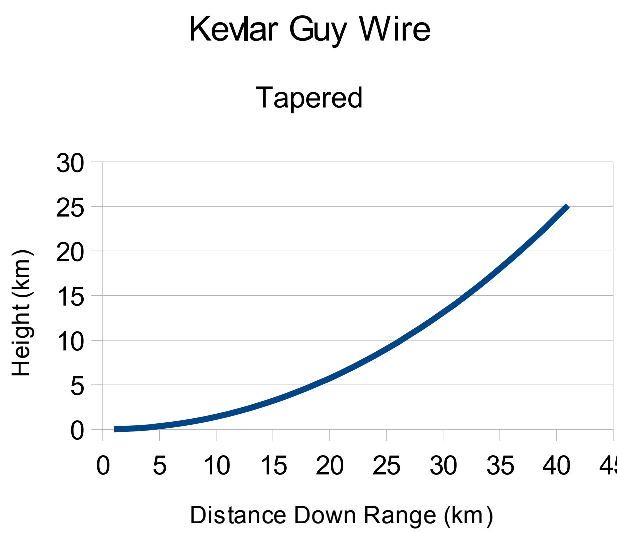

Figure B4. Tapered Kevlar Cable

In the proof of concept design, each 1.5 km section of the tower has four guy wires. Construction of the guy wire starts at the top of each section by adding thin layers to the cable one at a time, similar to the construction of the ribbon for the Space Track Launch System. The first set of guy wires begin at 1.5 km, the second set at 3.0 km, and so on until reaching 25 km.

At the top of the tower, the maximum vertical load on the tower from one guy wire is about 6.0×10^6 N. There are four guy wires at the top of the tower resulting in a total maximum dynamic load of 2.4×10^7 N. The static load on the tower resulting from the transfer station, rotating truss, ribbons, and counterweights is about 4.0×10^6 N. Therefore, the static load and the peak vertical load result in a peak dynamic load of 2.8×10^7 N on the tower supports. This is about 5 times the peak dynamic load in the proof of concept paper (Fisher, J. F., 2012). As such, a slight redesign of the proof of concept system is required if Kevlar is used for the cable.

If the Spectra® 2000 fiber is used for the cable, the results are slightly better. Using the same procedure with a Spectra® 2000 cable gives the results shown in figure B5 below.

At a height of 25 km, the tension in the cable is about 1.1×10^7 N and the vertical component of the tension is 8.6×10^6 N. At the anchor point, the cable is 1.7 cm thick and has an area of 8.3×10^{-3} m^2. It is 50 cm wide and has a mass of about 462 metric tons. At the top of the tower, the cable is 1.8 cm thick and has an area of 9.1×10^{-3} m^2. It makes a 24° angle with the tower and has a down range distance of 55 km. The transfer station, rotating truss, counterweights, and four guy wires result in a peak dynamic load of 2.2×10^7 N. Slightly better but still it is necessary to beef up the proof of concept design.

The down range distance of both the Kevlar and Spectra® cable can be reduced by anchoring the cable at a shorter distance from the tower and applying a corresponding vertical load. Anchoring the cable at a shorter distance from the tower may require a thicker and thus, a heavier cable. Tradeoff studies are required to determine the optimum design.

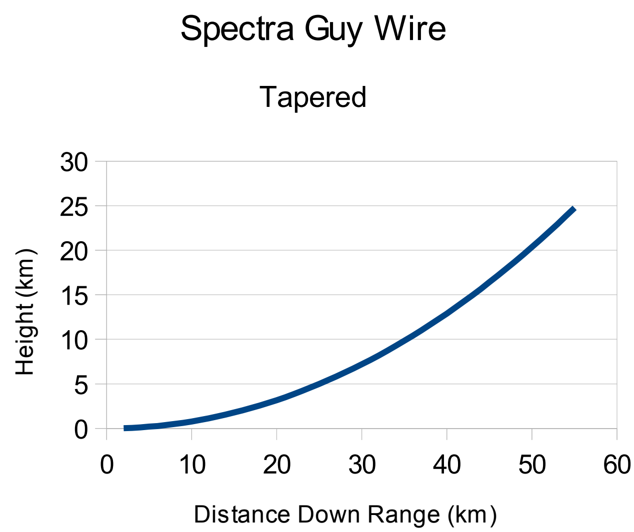

Figure B5. Tapered Spectra Cable

The first generation STLS tower is from 50-100 km high and the second generation tower is from 100-150 km high. To reach 150 km (figure B6), a 50 cm wide Spectra® cable would have to be 2.6 cm thick at the top, have a mass of about 1,200 metric tons, and a down range distance of 130 km. At the top of the tower, the cable would make a 50° angle with the vertical.

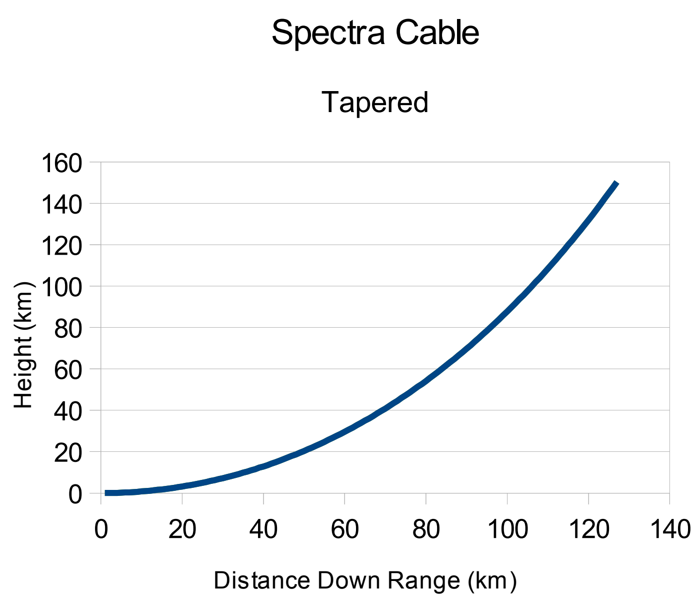

Figure B6. 150 km Tapered Spectra Cable

Hopefully, material properties will improve and become readily available before the final design phase of the first or second generation Space Track Launch System. A factor of 3 increase (M5), an order of magnitude increase (graphene), or 2 orders of magnitude increase (carbon nanotubes) in tensile strength would have a corresponding impact on the vertical component of the load. As such, the peak dynamic load on the tower would decrease resulting in a smaller number of inflatable beams required to support the transfer station, rotating truss, ribbons, and counterweights.

B5.0 Conclusion

Addendum B has shown that constant area guy wires are impractical for tall towers. The solution is to taper the cable from top to bottom. A Kevlar 49 cable can theoretically meet the 25 km height requirement of the proof of concept tower. Also, a Spectra 2000 cable can theoretically meet the 50-150 km height requirement for the first and second generation Space Track Launch System. Newer materials could drastically decrease the vertical component of the tension reducing the peak load on the tower supports.

References

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