Space Track Launch System
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by
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1. Introduction

Perhaps the first use of a rotating tether to sling payloads into orbit was mentioned by Joseph A. Carroll (Carroll, J.A.,1986). Carroll suggested that a rotating sling on the surface of an airless body such as the moon might accelerate 10-20 kg payloads to orbital velocity. J. Puig-Suari, et el, (Puig-Suari, J., Longuski, J. M., & Tragesser, S. G., 1995) explored the technical feasibility of slinging small payloads from the surface of the moon. The requirements for the tapered tether and the energy demands for some representative tether sling facilities were evaluated. In a NASA publication, D.V. Smitherman, Jr. (Smitherman Jr., D. V., 2000) suggested using a spinning tether on top of a 50 km tall tower as a sling to launch cargo into space from earth. A major drawback to this concept is the lateral force placed on the tower when the payload is released. The Space Track Launch System is similar to these concepts but with a few exceptions. Space Track uses a counterweight permanently attached to the end of the ribbon and rotational kinetic energy to launch the payload from the ribbon. In this manner, the launch velocity is greater and the launch load is transferred to the ribbon, keeping the dynamic load on the tower to a minimum. The ribbon can be inspected periodically and replaced if necessary.

The proposed Space Track Launch System is illustrated in Figure 1 below. Two tapered ribbons each of length l_r and cross sectional area A_x are attached to counter-weights (CW) with mass m_{cw}. A cargo pod (CP) travels down the length of the ribbon and launches off the ribbon at the counterweight. The cargo pod achieves a resultant velocity due to the gravitational, tangential, and centrifugal acceleration down the ribbon. The tower height is such that there is the smallest possible drag on the ribbon and tower.

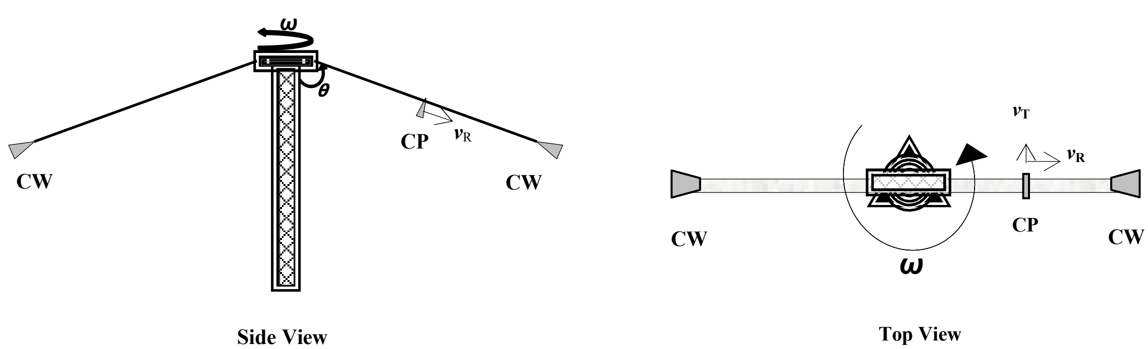

Figure 1. Space Track Launch System

In the following sections, the necessary equations are developed to determine the taper, area, and mass of the ribbon; the center of mass of the ribbon and counter-weight combination; the angular velocity; the resultant velocity of the cargo pod at launch; the angular momentum; and the rotational kinetic energy for the Space Track Launch System. With these equations, a possible launch system using presently available materials is evaluated.

2. Tapered Ribbon

The ribbon material, the tower height, and the launch altitude will determine the length of the ribbon and the angle it makes with the vertical. For an earth based launch system, atmospheric properties (mainly drag) force the tower height and launch altitude into the upper atmosphere. Although in some cases a uniform ribbon would work, a tapered ribbon is suggested because of the lengths involved.

The Space Track concept is similar to a conical pendulum. The pendulum (Figure 2) swings around an axis at an angle θ from the vertical. For a counterweight of mass m_{CW} revolving around an axis with radius r, the tension in the ribbon at the counterweight is given by,

$$T\cos\theta = m_{CW}\,g \qquad (1)$$

$$T\sin\theta = m_{CW}\,a_c \qquad (2)$$

where a_c is the centripetal acceleration, T is the tension in the ribbon, and g is the acceleration due to gravity.

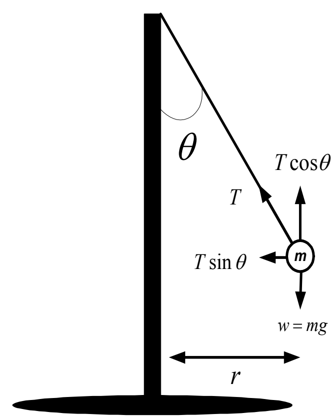

Figure 2. Conical Pendulum

For the Space Track Launch System, the width of the ribbon is kept constant to serve as a track for the cargo pod. Therefore, the ribbon will taper in thickness from the rotating hub to the counterweight. From equation 1, the tension at the counterweight is,

$$T_{CW} = \sigma\,A_{CW} = \frac{m_{CW}\,g}{\cos\theta}$$

where σ is the working tensile strength and A_{CW} is the area of the ribbon at the counterweight.

The area of the ribbon at the counterweight is equal to the greatest allowable stress afforded by the ribbon. Therefore,

$$A_{CW} = \frac{m_{CW}\, g}{\sigma \cos\theta} \qquad (3)$$

For a ribbon of length l_r, the cross sectional area of the ribbon increases from the counterweight to the rotating hub. The area at any point x along the ribbon can be determined by integrating from the rotating hub to the counterweight. Therefore,

$$A_x = \int_x^{l_r} \frac{g}{\sigma\cos\theta}\ dm_y + \frac{m_{CW}\, g}{\sigma\cos\theta}$$

The gravitational acceleration g varies slightly between the launch point and rotating hub. As such, g should be made a function of x. However, because g varies slightly, the integral is much simpler if g is kept constant. The higher value for g at the operating altitude will be used, which results in an added safety factor in the design. Substituting $\rho A_y\, dy$ for dm_y gives,

$$A_x = \int_x^{l_r} \frac{\rho g}{\sigma\cos\theta} A_y\, dy + \frac{m_{CW}\ g}{\sigma\cos\theta}$$

where ρ is the density of the material.

Taking the derivative with respect to *x* gives,

$$\frac{dA_x}{dx} = \frac{\rho g}{\sigma\cos\theta}\frac{d}{dx}\int_x^{l_r} A_y\, dy$$

$$= \frac{-\rho g}{\sigma\cos\theta} A_x$$

To meet the boundary condition at l_r (i.e. A_x is equal to A_{CW} at l_r), a possible solution for A_x is,

$$A_x = A_{CW} \exp\left[\left(\frac{\rho g}{\sigma\cos\theta}\right)(l_r - x)\right] \qquad (4)$$

Taking the derivative with respect to *x* will show that equation 4 is a solution for A_x.

The first term in the exponential is a constant. With the exception of the cos θ term, it is the inverse of the characteristic length l_c for the ribbon material. Therefore,

$$A_x = A_{CW} \exp\left(\frac{l_r - x}{l_c \cos\theta}\right) \qquad (5)$$

where

$$l_c = \frac{\sigma}{\rho g}$$

Once we pick our candidate material, determine the length of the ribbon, and the angle from the vertical, the cross sectional area of the ribbon can be determined at any point along the ribbon.

The mass m_r of the ribbon is given by,

$$m_r = \int_0^{l_r} \rho A_x \, dx$$

Substituting for A_x from equation 5 and evaluating the integral results in the mass ratio,

$$\frac{m_r}{m_{CW}} = \exp\left(\frac{l_r}{l_c \cos\theta}\right) - 1 \qquad (6)$$

See http://members.cox.net/fisherspacesystems/Equation6.pdf for a more complete derivation of equation 6.

3. Center of Mass

For $l_r > l_c \cos\theta$, the ribbon mass is several times greater than the counterweight mass. Therefore, the true center of mass of the ribbon and counterweight combination must be determined before the angular velocity can be derived.

The center of mass of the ribbon l_{cmr} is given by,

$$l_{cmr} = \frac{\int_0^{l_r} \rho A_x x \, dx}{\int_0^{l_r} \rho A_x \, dx}$$

Substituting A_x from equation 5 gives,

$$l_{cmr} = \frac{\int_0^{l_r} x \exp\left(\frac{-x}{l_c \cos\theta}\right) dx}{\int_0^{l_r} \exp\left(\frac{-x}{l_c \cos\theta}\right) dx}$$

Using integration by parts and substituting in the mass ratio from equation 4 gives,

$$l_{cmr} = l_c \cos\theta - l_r \left(\frac{m_{CW}}{m_r}\right)$$

where l_{cmr} is the center of mass for the ribbon.

The center of mass for the ribbon and counterweight combination is given by,

$$l_{cm} = \frac{m_r l_{cmr} + m_{CW} l_r}{m_r + m_{CW}} = \frac{\left(\frac{m_r}{m_{cw}}\right) l_{cmr} + l_r}{1 + \frac{m_r}{m_{CW}}} \qquad (7)$$

where the equation to the far right is in terms of the mass ratio.

See http://members.cox.net/fisherspacesystems/Equation7.pdf for a more complete derivation of equation 7.

From equations 1 and 2, the angular velocity ω can be derived in terms of the distance to the center of mass l_{cm} and the angle from the vertical θ. Dividing $T \sin\theta$ by $T \cos\theta$ gives,

$$\omega = \sqrt{\frac{g}{l_{cm} \cos\theta}} \qquad (8)$$

The velocity of the cargo pod at launch v_L can now be determined.

4. Launch Velocity

It can be seen from Figure 1 that the velocity of the cargo pod at launch is,

$$v_L = \sqrt{v_T^2 + v_R^2} \qquad (9)$$

where v_T is the tangential velocity and v_R is the radial velocity of the cargo pod.

Figure 1 shows a cargo pod (CP) accelerating down the ribbon. The tangential velocity of the cargo pod at the end of the ribbon (the launch point) is,

$$v_T = \omega\, l_r \sin\theta \qquad (10)$$

The work-energy relationship is used to determine the radial velocity v_R.

The radial force F_R on the cargo pod at any point x along the ribbon is equal to the vector sum of the centrifugal force and the gravitational force at x. Therefore,

$$F_R = \sqrt{\left(m_{CP}\,\omega^2 x \sin\theta\right)^2 + \left(m_{CP}\, g\right)^2}$$

For long ribbons, the force due to gravity is small compared to the centrifugal force. Therefore, the radial force can be approximated by dropping the gravity term. This gives,

$$F_R \approx m_{CP}\omega^2 x \sin\theta$$

The work done on the cargo pod from the rotating hub to the launch point is,

$$W = \int_0^{l_r} F_R \, dx$$

Substituting F_R from above and integrating gives,

$$W = \frac{1}{2} m_{CP}\, \omega^2 l_r^2 \sin\theta$$

The work is equal to the change in kinetic energy of the cargo pod in the radial direction at launch. Therefore,

$$\frac{1}{2} m_{CP} v_R^2 = \frac{1}{2} m_{CP}\omega^2 l_r^2 \sin\theta$$

Solving for v_R gives,

$$v_R = \omega l_r \sqrt{\sin\theta} \qquad (11)$$

Substituting v_T from equation 10 and v_R from equation 11 into equation 9 gives,

$$v_L = \omega l_r \sqrt{\sin\theta\left(1 + \sin\theta\right)} \qquad (12)$$

where v_L is the velocity of the cargo pod at launch.

5. Moment of Inertia

To complete the evaluation, the angular momentum L and the Kinetic Energy of Rotation (*K.E.R.*) is needed. To determine *L* and *K.E.R*., the total moment of inertia of the ribbon and counterweight combination is calculated. The moment of inertia for the ribbon is given by,

$$I_r = \int_0^{l_r} x^2 \sin^2\theta \rho A_x \, dx$$

Substituting for A_x from equation 5 and using integration by parts gives,

$$I_r = 2m_{CW}\left(l_c \cos\theta \sin\theta\right)^2 \left[\exp\left(\frac{l_r}{l_c \cos\theta}\right) - \frac{1}{2}\left(\frac{l_r}{l_c \cos\theta}\right)^2 - \left(\frac{l_r}{l_c \cos\theta}\right) - 1\right] \qquad (13)$$

Equation 13 is the moment of inertia for just the ribbon.

See http://members.cox.net/fisherspacesystems/Equation13.pdf for a more complete derivation of equation 13.

The moment of inertia for a point mass revolving around a vertical axis is mr^2. Therefore, the moment of inertia for the counterweight is,

$$I_{CW} = m_{CW}\left(l_r \sin\theta\right)^2 \qquad (14)$$

Since there are two ribbons and two counterweights, the total moment of inertia I_t for the system is equal to 2(I_r + I_{CW}). Therefore, the angular momentum and the *K.E.R.* of the Space Track Launch System is,

$$L = I_t\omega \quad \text{and} \quad K.E.R. = \frac{1}{2} I_t \omega^2$$

6. Space Track Launch System

With the necessary equations in hand, a possible first generation launch system can be explored. For example, in the initial analysis a tower height of 150 km is chosen. With a tower height of 150 km, there will be no collisions with existing Earth orbiting satellites. Also, the tower, ribbon, and cargo pod are above the atmosphere and will experience little drag. However, orbital debris and environmental threats to the system exist. All of these factors must be considered when determining the final tower height and launch altitude.

The first task is to select a ribbon material. There are three possible candidate materials to choose from; Spectra® 2000, Vectran®, and a new material called M5®. The table below compares the characteristic length of several presently available fibers (Vectran®, 2006; Spectra®, 2006; and M5®, 2006).

Material	Density (kg/m^3)	Tensile Strength (GPa)	Characteristic Length (km)
Vectran® Fiber	1400	3.2	233
Spectra® 2000	970	3.5	368
M5® (Tested)	1700	5.5	330
M5® (Targeted)	1700	10.0	600

Presently, Spectra® 2000 has the greater characteristic length and therefore, is the material of choice. However, if Magellan International and Dupont succeed in reaching the targeted tensile strength, then M5® has the greater length and the launch parameters will improve. Both Spectra® 2000 and M5® have the added advantage of being resistant to UV radiation, an attractive feature in the upper atmosphere.

The second task is to determine the ribbon length. There are several factors to consider to make Space Track a viable launch system. First, there must be enough initial velocity at a reasonable altitude to get a 10 ton cargo pod with a reasonable payload into LEO. Second, the ribbon and counterweight must have enough moment of inertia (rotating mass) to transfer energy to the cargo pod. The moment of inertia must be large enough to keep the counterweight and ribbon from entering the upper atmosphere once the cargo pod launches from the ribbon.

Third, the electric motors must have enough power to restore the *K.E.R.* to the ribbon after launch in a reasonable amount of time.

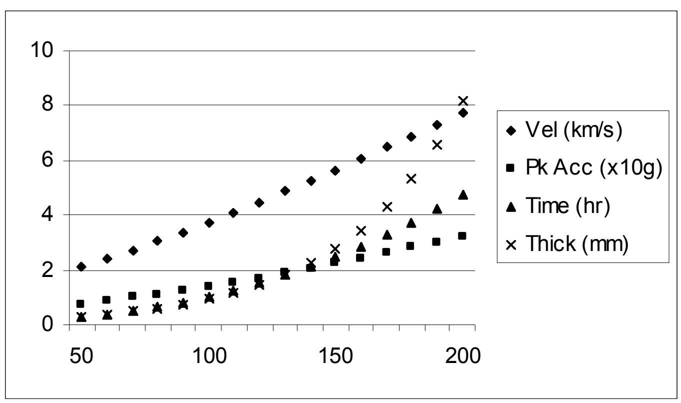

Figure 3. Launch Parameters for Spectra® Ribbon

Figure 3 shows a plot of launch velocity, peak acceleration, time to restore the *K.E.R.,* and the thickness of the ribbon at the axis of rotation. The launch parameters are given for a ribbon from 50 km to 200 km in length. The ribbon is 2 m wide from the rotating hub to the counterweight. From the analysis, a ribbon 180 km long at an angle of 75.5^o from the vertical and 5.3 mm thick at the hub seems reasonable. With a 1.75 GPa working tensile strength and a 10 ton counterweight, the moment of inertia of the system is 3.6 x 10^{15} kg-m^2 and the angular velocity is 2.9 x 10^{-2} rad/sec. With three, 7,000 hp electric motors (1.6 x 10^7 J/s), it will take about one day to build up the initial *K.E.R*.

Once Space Track has the initial *K.E.R.*, an elevator carries a 10 ton cargo pod to the top of the tower. The cargo pod attaches to the ribbon and begins to accelerate. At the counterweight, the cargo pod launches from the ribbon. The launch velocity is approximately 6.9 km/s, the acceleration is about 29 g, and the time to restore the *K.E.R*. is around 3.8 hours. For the tower height, angle, and length used in this example, the counterweight is at an initial altitude of 105 km. The angle the ribbon makes with the vertical decreases slightly to 73.9^o resulting in a launch altitude of 102 km.

From the ideal rocket equation, the empty mass m_e delivered to a LEO orbit can be estimated. The ideal rocket equation is,

$$\Delta u = I_{sp}\, g \ln\left(\frac{m_{CP}}{m_e}\right)$$

where Δu is the change in velocity required for a 400 km orbit (~1.1 km/s), I_{sp} is the specific impulse (~290 sec), and g is the local gravitational acceleration (~9.5 m/s^2 at 100 km).

Solving for m_e and substituting in the values for Δu, I_{sp}, and g above gives an empty mass of approximately 6.8 ton. The cargo pod structure, guidance and control, and orbital maneuvering system take up an estimated 2.0 ton. This leaves about 4.8 ton for payload. If there are two elevators in the tower and they can travel at 30 km/hr, another 10 ton payload could be delivered around the same time that the *K.E.R*. is restored. Therefore, Space Track should be able to deliver a 4.8 ton payload to LEO every 5 hours.

If the cargo pod uses cryogenic fuel and oxidizer with a vacuum specific impulse of 450 sec, the payload delivered to orbit increases to about 5.8 ton. Also, if a spacecraft launches from the ribbon at around the 32 km point from the hub, the peak acceleration will be 6 g. At this point, the launch altitude will be approximately 142 km at a velocity of 1.6 km/s. With a 10 ton spacecraft, a 235 kg payload can be delivered to LEO. This is enough mass for two people plus supplies. Therefore, the Space Track Launch System could start a whole new market in general aviation, a market for privately owned and operated recreational spacecraft.

If Magellan and Dupont achieve the target tensile strength of 10 GPa for the M5® material, launch parameters will improve. Figure 4 below shows the relationship between the launch velocity of the cargo pod, peak acceleration, time to restore *K.E.R*., and the thickness at the hub for an M5® ribbon 2 m wide.

With a 5 GPa working tensile strength, the moment of inertia of the system is 4.8 x 10^{15} kg-m^2 and the angular velocity is 2.8 x 10^{-2} rad/sec. With 1.6 x 10^7 J/s available from the electric motors, it will take about 1.5 days to build up the *K.E.R*. The launch velocity for a cargo pod from the ribbon is approximately 8.3 km/s, the acceleration is about 33 g, and the time to restore the *K.E.R.* is around 5.5 hours.

The initial angle the ribbon makes with the vertical is about 78.5^o. This drops to about 77^o at launch resulting in a launch altitude of 104 km. At this altitude, it takes around 8 km/s to reach a 400 km orbit. With a little guidance and control, a 10 ton cargo pod could get an 8 ton payload to LEO. With a M5® ribbon, it may be possible for Space Track to launch a 10 ton cargo pod with an 8.0 ton payload every 6 hours.

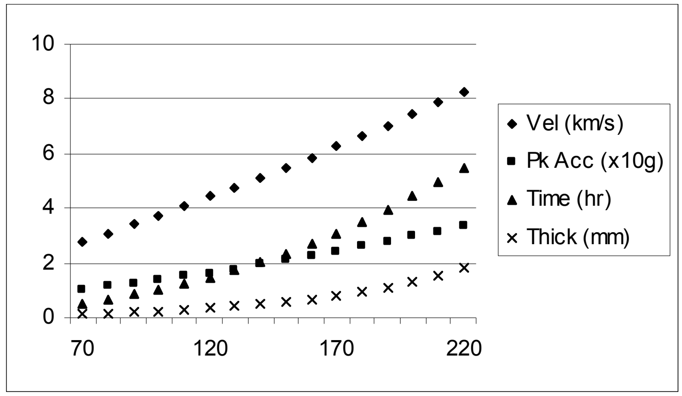

Figure 4. Launch Parameters for M5® Ribbon

If a manned spacecraft using cryogenic fuel and oxidizer launches from the ribbon at around the 33 km point from the hub, the peak acceleration is 6 g. At this point, the launch altitude is approximately 143 km at a velocity of 1.6 km/s. With a 10 ton spacecraft, a 243 kg payload can be delivered to LEO. This is enough mass for two people plus supplies.

What about a carbon nanotube (CNT) ribbon? Recent literature (Wei, C., Cho, K., & Srivastava, D., 2003) has suggested that 50 GPa CNT fiber is possible in the near future. Would there be any advantage using a stronger ribbon with a characteristic length of 3,925 km?

Using a working tensile strength of 25 GPa with a density of 1300 kg/m^3, possible launch parameters can be evaluated. Figure 5 shows the relationship between launch velocity, peak acceleration, time it takes to restore the *K.E.R.*, and ribbon thickness at the hub for a 10 ton spacecraft as a function of CNT ribbon length.
For a ribbon length of 220 km, the resultant velocity at launch is about 5.3 km/s with a peak acceleration of about 11 g. With two 220 km CNT ribbons and two 50 ton counterweights (50 ton is needed for the angular momentum), the moment of inertia for the system is 5.6 x 10^{15} kg-m^2 and the *K.E.R.* is 7.5 x 10^{11} J. It would take the electric motors about 13 hours to build up the initial *K.E.R.* and a little over 2 hours to restore the lost *K.E. R.* after launch.

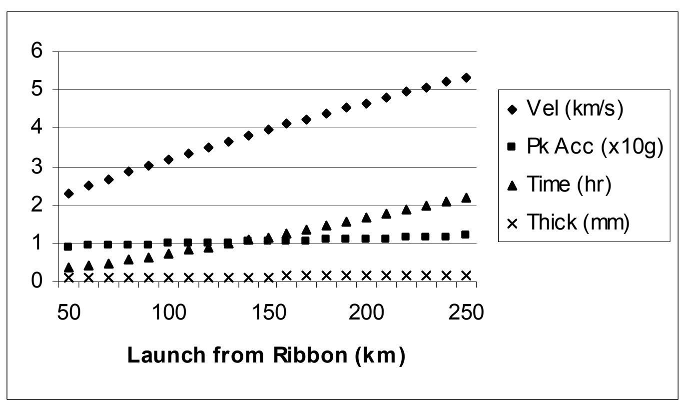

Figure 5. Launch Parameters for CNT Ribbon

The initial angle the ribbon makes with the vertical is 78° and drops to 76.3° at launch. This gives a launch altitude of around 101 km. With a 10 ton spacecraft and a launch velocity of 4.9 km/s, the Space Track Launch System could launch approximately 1.3 ton of payload using solid rocket motors and 2.9 ton of payload using cryogenic fuel and oxidizer.

If the spacecraft launches at the 6 g point, the launch velocity is approximately 1.9 km/s at an altitude of 144 km. With cryogenics, Space Track could transfer around 409 kg into a 400 km orbit. This is enough mass for one pilot and several passengers.

Therefore, the only real advantage of a CNT ribbon over a Spectra® 2000 or M5® ribbon is the ability to launch more people into space. Cargo and raw materials can be launched using presently available fibers.

The above analysis assumes ideal conditions and no friction or drag losses. These losses, and others, must be included in the Phase I evaluation of the Space Track Launch System. However, assuming a 10 ton cargo pod could be launched once a day, the Space Track Launch System could still revolutionize space travel.

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